

Specific Angular Momentum of Extrasolar Planetary Systems

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ABSTRACT

Angular momentum in our solar system is largely distributed between the Sun's rotation and the planetary orbits, with most of it residing in the orbital angular momentum of Jupiter. By treating the solar system as a two body central potential between the Sun and Jupiter, one can show that the orbital specific angular momentum of the two-body system exceeds the solar rotational specific angular momentum by nearly two orders of magnitude. We extend this analysis to the known extrasolar planets available in the Extrasolar Planet Encyclopedia and estimate the partitioning of each system's angular momentum into orbital and rotational components, ignoring the spin angular momentum of the planets. We find the range of partitioning of specific angular momentum in these systems to be large, with some systems near the stellar rotational limit, and others with orbital specific angular momentum exceeding this limit by three orders of magnitude. Planets in systems with high specific angular momentum have masses greater than two Jupiter masses, while those in systems with low specific angular momentum are below two Jupiter masses. This leads to the conclusion that low mass planets lose angular momentum more efficiently, and are thus more prone to migration, than larger mass planets.

Subject headings: extrasolar planets

1. Introduction

In 2006, Butler *et al.* published a catalog of 172 known low-mass companions. As of August 1st, 2007, 248 extrasolar planets, including 25 multi-planet systems, are cataloged in the online Extrasolar Planet Encyclopedia (Schneider 2007). With such an extensive catalog, it is possible to perform a meta-analysis of these objects to uncover some of the general properties of solar system formation. Previous studies have already analyzed the distribution of masses, eccentricities, orbital distances (Butler *et al.* 2006; Halbwachs *et al.* 2005) and stellar properties (Gonzalez and Laws 2007; Eggenberger *et al.* 2004; Santos *et al.* 2003). However,

a comprehensive study of the distribution of each system’s angular momentum has yet to be performed.

One notable feature of our solar system with implications for solar system formation is the distribution of angular momentum. Much of the angular momentum in our solar system is in the orbital motion of Jupiter. Treating Jupiter and the Sun as massive objects orbiting the system’s center of mass on circular orbits, the specific orbital angular momentum of the Sun (that is, the orbital angular momentum divided by the Sun’s mass) is approximately $1 \times 10^{10} \text{ m}^2 \text{ s}^{-1}$, which is dwarfed by the specific orbital angular momentum of Jupiter (the orbital angular momentum divided by Jupiter’s mass), $1 \times 10^{16} \text{ m}^2 \text{ s}^{-1}$. As we will explore later, the system’s total orbital specific angular momentum, largely due to Jupiter, is two orders of magnitude larger than the spin angular momentum of the Sun.

By extending the study of angular momentum to other extrasolar planets, we can examine the nature of these systems and probe their formation mechanisms. Similar work has been performed on the combined orbital properties of the extrasolar planets. For example, Udry *et al.* (2003) examined the distribution of orbital periods and masses of the extrasolar planets as they relate to planet migration theory. They discovered that the so-called “Hot Jupiters” tended to be lower mass planets compared to planets more distant from their central stars (see Figure 1. in Udry *et al.*), and also noticed several gaps in the period distribution indicating that planet migration is mass and distance dependent. The goal of this study is to explore these ideas in more detail, by comparing the *specific angular momentum* of planets across systems, thus removing the mass dependence.

Casting the problem in the light of a two-body central potential for those systems with only one known planet, we can estimate the orbital angular momentum for each system (estimating the angular momentum of the multi-planet systems requires more detailed numerical calculations, which are the subject of a later paper). Since most of the planets have been discovered using the radial velocity technique (Butler *et al.* 2006), we have the minimum mass, $M_p \sin(i)$, the orbital properties (semimajor axis, eccentricity, and orbital period) as well as the properties of the star (star type and mass, and an estimate of the star’s radius). In §2, we describe the catalog and our selection of systems to use for this study. In §3, we discuss the angular momentum calculations, both the stellar rotational component and the orbital component, in §4 we show the distribution of angular momentum with respect to mass, stellar mass, eccentricity, and planetary semi-major axis, with the implications of this work outlined in §5.

2. Angular momentum calculation

For this study, we took a sample of 145 planetary systems that were currently known to consist of only one star and one planet. From this sample we removed any planet that contained incomplete data for properties such as mass, orbital semi-major axis, orbital eccentricity and the mass of the parent star.

2.1. Orbital angular momentum

Treating each planet-star system as a two-body central force problem we calculated the system’s total orbital angular momentum,

$$L = \mu \sqrt{GMa(1 - e^2)}, \quad (1)$$

where μ is the reduced mass, G is the universal gravitational constant, M is the combined stellar and planetary mass, a is the semi-major axis and e is the eccentricity. This provides the total orbital angular momentum only, excluding any rotational angular momentum of the parent star. The specific orbital angular momentum of the system is thus L/M .

2.2. Stellar rotational angular momentum

To compute the stellar rotational angular momentum, we calculated the rotational angular momentum for solid, uniform density spheres with the masses, radii, and equatorial velocities reported in Lang (1992) for G0 through O8 stars. To compensate for the fact the stars are not solid spheres, we used the moment of inertia of the Sun calculated from solar models ($0.073 M_{\odot} R_{\odot}^2$, Guzik (2007)) to compute a scale factor for the mass distribution of the star compared to the solid approximation. Thus, the rotational angular momentum of the star, $L_{*,rot}$, is given by

$$L_{*,rot} = \eta I_{*,solid} \frac{v_{rot}}{R_*}, \quad (2)$$

where v_{rot} is the equatorial rotation of the star as given in Lang (1992), R_* is the radius of the star, $I_{*,solid}$ is the solid body momentum of inertia for a sphere,

$$I_{*,solid} = \frac{2}{5} M_* R_*^2, \quad (3)$$

and η is the scale factor determined by

$$\eta = \frac{I_{\odot}}{I_{\odot, \text{solid}}}, \quad (4)$$

with $I_{\odot, \text{solid}}$ the solid body moment of inertia for the Sun and $I_{\odot} = 0.073 M_{\odot} R_{\odot}^2$.

The rotational values for the stars in our sample are modeled with a best fit line (shown in Figure 2). For each of the planet-star systems in the catalog, we add the stellar rotational angular momentum according to the following formulae,

$$\log \frac{L_{*, \text{rot}}}{M} = 5.3246 \log \frac{M_*}{M_{\odot}} + 11.57 \quad (5)$$

for stars with masses less than $2.0 M_{\odot}$ and

$$\log \frac{L_{*, \text{rot}}}{M} = 0.7277 \log \frac{M_*}{M_{\odot}} + 13.015 \quad (6)$$

for stars with masses greater than $2.0 M_{\odot}$.

The stellar rotational angular momentum is added to the orbital angular momentum of the star-planet system to compute the total angular momentum of the system

$$L_{\text{tot}} = L + L_{*, \text{rot}}. \quad (7)$$

This is divided by the total system mass to get the specific angular momentum, L_{tot}/M , in units of $m^2 s^{-1}$.

3. Results

The full range of the specific angular momenta of these systems spans three orders of magnitude, with some near the rotational limit, and others far above the contribution due to a Jupiter-like orbit. To further explore this, we generated histograms of the planetary properties for those systems with angular momenta above $10^{13} m^2 s^{-1}$ (corresponding to the specific angular momentum of the Jupiter-Sun system). Figure 1 shows the fraction of systems for the planet mass (panel A), the stellar mass (panel B), the eccentricity (panel C) and the planet semimajor axis (panel D). The solid line is the total sample, the dashed line

the system with specific angular momentum less than $10^{13} \text{ m}^2 \text{ s}^{-1}$, and the dotted line the systems with specific angular momentum greater than $10^{13} \text{ m}^2 \text{ s}^{-1}$.

According to Panel A in Figure 1, the lower mass planets in the sample (those below $2.0 M_J$) tend to have low values of the specific angular momenta. As expected, these also correspond to the systems with the smaller semimajor axis. However, there appears to be no pattern with respect to stellar mass, and only a slight preference for low angular momentum at lower eccentricity, associated with the circularized, close-in, low mass systems.

The total specific angular momenta are plotted in Figure 2 vs. the stellar mass. The solid squares are the specific rotational angular momenta for the G0 - O8 stars, and the solid straight lines the linear fits to those points, given by Eqns. (5) and (6). The circles are the total specific angular momentum of the system, solid circles for planets greater than $2.0 M_J$ and open circles for planets with mass less than $2.0 M_J$. In addition, we have added a curve representing the total specific angular momentum of a system with a Jupiter-mass planet orbiting a Sun-like star at the distance of 5.2 A.U. Again, larger mass planets tend to have a higher specific angular momentum, above the Jupiter-Sun line. This is seen more clearly in Figure 3, which shows the full 2-D number histogram for specific angular momentum against planet mass. We see a clear tendency of higher specific angular momentum with planet mass.

4. Discussion

The analysis outline above leads us to the following observations:

- Planets with masses less than $2.0 M_J$ have the lowest total specific angular momentum.
- Planets with masses greater than $2.0 M_J$ have high total specific angular momentum.
- Close-in planets also tend to have low total specific angular momentum.
- There is little dependence of specific angular momentum on eccentricity and stellar mass

These observations, combined with previous work on the mass dependence of planetary migration (Udry *et al.* 2003), show the low mass “Hot Jupiters” lost large amounts of angular momentum compared to their high mass counterparts, which do not undergo migration. The mass cutoff for this migration appears to be $2 M_J$, with low mass systems losing $100 \text{ m}^2 \text{ s}^{-1}$ to $1000 \text{ m}^2 \text{ s}^{-1}$ of specific angular momentum during migration.

Observational biases exist in the extrasolar planet database. The inclination of the system, unknown in the vast majority of cases, will tend to increase our calculated values of the the total angular momentum. However, systems inclined 45 degrees would only increase the angular momentum by 42 %, which is small considering the angular momentum ranges over three orders of magnitude. In addition, after 11 years of searching, observers are just now probing Jupiter-mass companions at Jupiter-like distances. However, the “Hot Jupiters” of the extrasolar planet parameter space is the most well sampled and is most sensitive to high-mass planets. If there were close-in, high-mass planets that were subject to angular momentum loss through migration, we would see them in this sample.

There are a number of additional questions to be addressed following this initial study. As it stands, existing, undiscovered planets in these 145 systems might account for some of the “missing” angular momentum. A detailed study of the multi-planet systems will shed light on this issue. A follow-up paper will study the angular momentum of the multi-planet systems, using N-body calculation to constrain the partitioning of momentum in the system. Using the systems from the Extrasolar Encyclopedia, we will perform Monte Carlo simulations, varying orbital parameters to search the parameter space for likely distributions of angular momentum partitioning in the systems.

We are grateful to Brad Carroll and Dale Ostlie for Problem 2.6 in *Modern Astrophysics* (Carroll and Ostlie 2006) which was the motivation behind this work and the original inspiration for Figure 2, which was based on Figure 23.8 on page 858 of their text. We also thank Joyce Guzik for her assistance with the solar model. We would also like to thank the 2006 Weber State University Astrophysics class who willingly extended this problem to cover the known extrasolar planets (for extra credit, of course).

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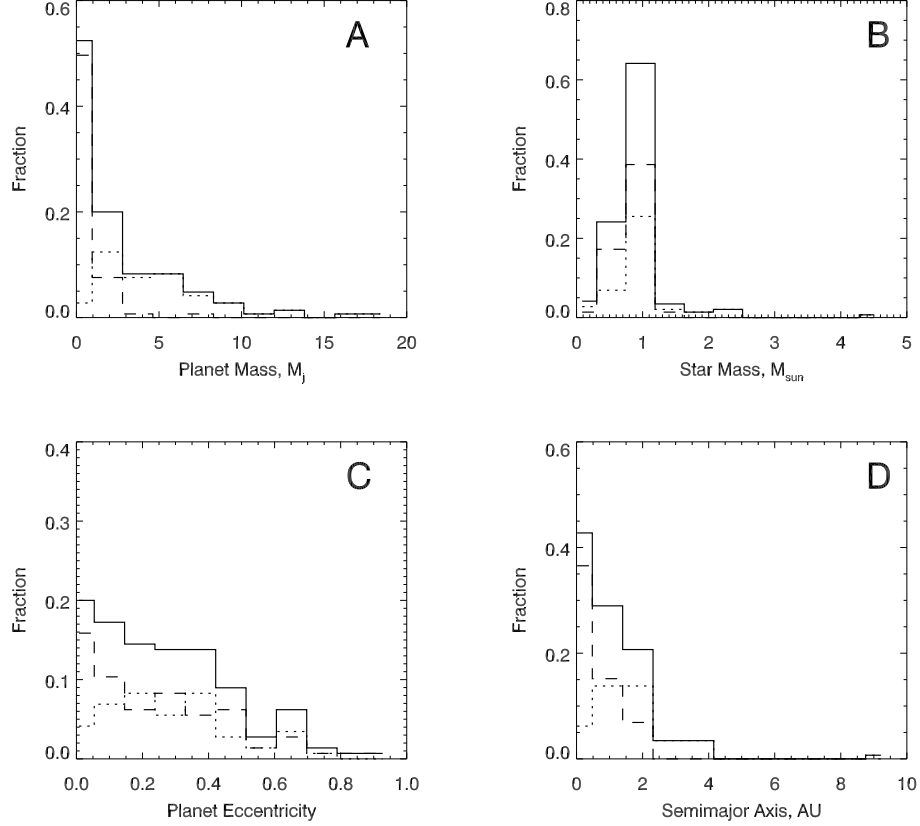


Fig. 1.— Histogram showing the fraction of the total systems with measured parameters for the planets and stars in the sample. Panel A is the planet mass (in units of M_J), panel B is the stellar mass (in units of M_{\odot}), Panel C is the eccentricity, and panel D is the semimajor axis (in AU). The solid lines represent all the objects in the sample, the dashed lines are those systems with a specific angular momentum less than $10^{13} \text{ m}^2 \text{ s}^{-1}$, and the dotted lines are those systems with a specific angular momentum greater than $10^{13} \text{ m}^2 \text{ s}^{-1}$.

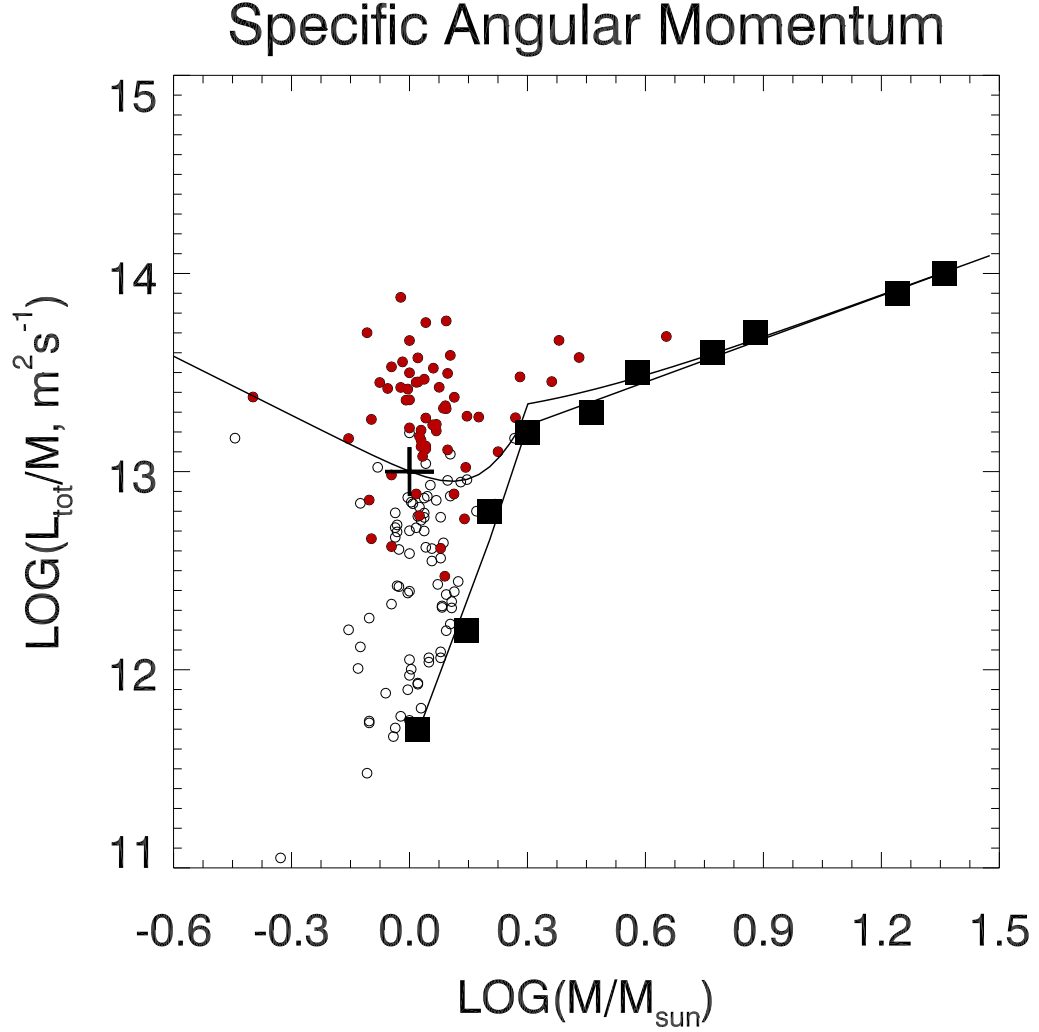


Fig. 2.— Plot of the specific angular momentum for systems with one known planet. The solid squares are the rotational angular momenta for main sequence stars G through O, with associated trendlines used in the calculations. The solid curve is the model of Jupiter orbiting a star of the given mass. The circles represent specific angular momentum of the 145 single planet systems. Filled circles are planets with mass greater than $2.0 M_J$ and the open circles are planets with mass less than $2.0 M_J$. The cross represents the Jupiter-Sun system.

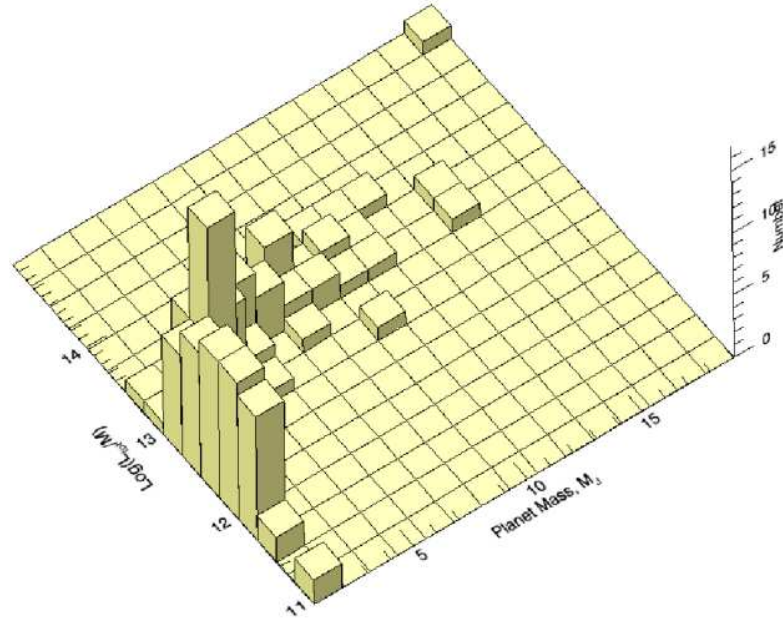


Fig. 3.— Two-dimensional histograms showing the number of systems with planetary mass for the planets in the sample vs. the log of the specific angular momentum (with units of $m^2 s^{-1}$).